Polynomiality of off-forward distribution functions in the chiral quark soliton model

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Abstract

Mellin moments of off-forward distribution functions are even polynomials of the skewedness parameter ξ. This constraint, called polynomiality property, follows from Lorentz-invariance. We prove that the unpolarized off-forward distribution functions in the chiral quark soliton model satisfy the polynomiality property. The proof is an important contribution to the demonstration that the description of off-forward distribution functions in the model is consistent. As a byproduct of the proof we derive explicit model expressions for moments of the D-term and compute the first coefficient in the Gegenbauer expansion for this term.

1 Introduction

Off-forward distribution functions (OFDFs) of partons in the nucleon – or equivalently double, skewed or non-diagonal parton distribution functions [1, 2, 3, 4] – are a promising source of new information on the internal nucleon structure (see Refs. [5, 6, 7] for recent reviews). OFDFs enter the description of hard exclusive processes, such as hard meson production [4] and deeply virtual Compton scattering [8]. Only recently first data on the latter process became available [9, 10, 11, 12], and so our present understanding of non-perturbative aspects of OFDFs has to rely on models.

After the first estimates in the framework of the MIT bag model [13], studies in the chiral quark soliton model (χQSM) of the nucleon have been presented [14, 15]. The χQSM has been derived from the instanton model of the QCD vacuum and provides a relativistic quantum field-theoretical description of the nucleon. Without any adjustable parameters it describes a large variety of nucleonic properties, like form factors [16, 17] and (anti)quark distribution functions [18, 19, 20] typically within $(10-30)\%$.

However, the reliability of a model is not only based on its phenomenological success. From a theoretical point of view, it is important to demonstrate the internal consistency of the model. One of the virtues of the χQSM, which is due to its field-theoretical character, is the possibility to provide analytical proofs which explicitly demonstrate the consistency of the model. In Ref. [18] it has been proven that the quark and antiquark distribution functions computed in the model satisfy all general requirements, such as sum rules, positivity and inequalities. With the same rigour it has been shown in Refs. [14, 15] that the χQSM expressions for OFDFs reduce to usual parton distributions in the forward limit, and that their first moments yield form factors.

In this paper we will present a further check of consistency for the χQSM, and give for the unpolarized OFDFs an explicit proof of polynomiality, i.e. of the property that the m^{th} Mellin moment of an OFDF is an even polynomial in the skewedness variable ξ of degree less than or equal to m at the (unphysical) value $t = 0$ of the Mandelstam variable[1]. The polynomiality

[1] Here we use the notation of Ref. [3].

property follows from Lorentz and time reversal invariance of strong interactions. The proof of polynomiality is the main result of this paper.

An elegant way to satisfy the polynomiality condition is to use the double distributions (DDs) of Refs. [1, 2]. However, if assumed to be regular functions, DDs yield a vanishing highest power of ξ for singlet moments of unpolarized OFDFs. This means that DDs yield an incomplete description, which can be completed by introducing the so-called D-term [21]. Moments of the D-term are uniquely defined in terms of the coefficients of the highest power of ξ [21, 22, 23]. In Ref. [24] the D-term has been calculated numerically from moments of the OFDFs evaluated in the model for physical values of t. The results obtained here enable us to evaluate in the model, in principle, all Mellin moments of the D-term directly at the unphysical point $t = 0$. We will demonstrate this for the lowest non-vanishing moment.

The paper is organized as follows. In Sec. 2 a brief introduction to the χQSM is given. In Sec. 3 the model expressions for the unpolarized OFDFs are presented. The proof of polynomiality is given in Secs. 4 and 5 for the non-spin-flip and the spin-flip OFDF. Sec. 6 is devoted to the D-term. Finally, in Sec. 7 we summarize our results and conclude. Some technical details required for the proofs are given in the Appendices.

2 The chiral quark soliton model (χQSM)

The χQSM [25] is essentially based on the principles of chiral symmetry breaking and the limit of a large number of colours N_c. The effective chiral relativistic quantum field theory, which underlies the χQSM, is formulated in terms of quark, antiquark (ψ, $\bar{\psi}$) and Goldstone boson (pion π^a, $a = 1, 2, 3$) degrees of freedom. It is given by the partition function [26, 27, 28]

$$Z_{\rm eff} = \int\mathcal{D}\psi\,\mathcal{D}\bar{\psi}\,\mathcal{D}U\;\exp\biggl(i\int {\rm d}^4x\;\bar{\psi}\,(i\partial\!\!\!/ - M\,U^{\gamma_5})\psi\biggr)\;,\quad U^{\gamma_5} = e^{i\gamma_5\tau^a\pi^a}. \tag{1}$$

In Eq. (1) M is the dynamical quark mass due to spontaneous breakdown of chiral symmetry, which is in general momentum dependent, and $U = \exp(i\tau^a\pi^a)$ denotes the $SU(2)$ chiral pion field. The effective theory in Eq. (1) contains the Wess-Zumino term and the four-derivative Gasser-Leutwyler terms with correct coefficients. It has been derived from the instanton model of the QCD vacuum [28, 29], which provides a mechanism of chiral symmetry breaking. The effective theory (1) is valid at low energies below 600 MeV, a scale set by the inverse of the average instanton size.

One application of the effective theory (1) is the chiral quark soliton model (χQSM) of baryons [25]. In large N_c limit the path integral over pion field configurations in Eq. (1) can be solved in the saddle-point approximation. The large N_c limit is known to be a good theoretical guideline. In this limit the nucleon can be viewed as a classical soliton of the pion field [30]. The χQSM provides a realization of this idea. In the leading order of the large N_c limit the pion field is static, and one can determine the spectrum of the effective one-particle Hamiltonian of the theory (1)

$$\hat{H}_{\rm eff}|n\rangle = E_n|n\rangle \;\;,\quad \hat{H}_{\rm eff} = -i\gamma^0\gamma^k\partial_k + \gamma^0 M U^{\gamma_5}\;. \tag{2}$$

The spectrum consists of an upper and a lower Dirac continuum, which are distorted by the pion field as compared to continua of the free Dirac-Hamiltonian

$$\hat{H}_0|n_0\rangle = E_{n_0}|n_0\rangle \;\;,\quad \hat{H}_0 = -i\gamma^0\gamma^k\partial_k + \gamma^0 M\;, \tag{3}$$

and of a discrete bound state level of energy $E_{\rm lev}$, for a strong enough pion field of unity winding number. By occupying the discrete level and the states of lower continuum each by N_c quarks in an anti-symmetric colour state, one obtains a state with unity baryon number. The soliton energy $E_{\rm sol}$ is a functional of the pion field,

$$E_{\rm sol}[U] = N_c\biggl(E_{\rm lev} + \sum_{E_n<0}(E_n - E_{n_0})\biggr)\;. \tag{4}$$

Minimization of $E_{\rm sol}[U]$ determines the self consistent solitonic pion field U_c. This procedure is performed for symmetry reasons in the so-called hedgehog ansatz $\pi^a(\mathbf{x}) = \frac{x^a}{|\mathbf{x}|}\,P(|\mathbf{x}|)$ with the radial (soliton profile) function $P(|\mathbf{x}|)$. The nucleon mass $M_{\rm N}$ is given by $E_{\rm sol}[U_c]$. Quantum numbers of the baryon like momentum, spin and isospin are described by considering zero modes of the soliton. Corrections in $1/N_c$ can be included by considering time dependent pion field configurations. The results of the χQSM respect all general counting rules of the large N_c phenomenology.

For the following it is important to note that the effective Hamiltonian $\hat{H}_{\rm eff}$ commutes with the parity operator $\hat{\Pi}$ and the grand-spin operator $\hat{\mathbf{K}}$, defined as the sum of quark angular momentum and isospin. Thus $\hat{H}_{\rm eff}$, $\hat{\Pi}$, $\hat{\mathbf{K}}^2$ and $\hat{K}^3$ form a maximal set of commuting operators, and the single quark states $|n\rangle$ in Eq. (2) are characterized by the quantum numbers parity π, K and M

$$|n\rangle = |E_n,\,\pi,\,K,\,M\rangle\;. \tag{5}$$

The χQSM allows to evaluate in a parameter-free way nucleon matrix elements of QCD quark bilinear operators as

$$\begin{aligned}&\langle N',\mathbf{P}'|\bar{\psi}(z_1)\Gamma\psi(z_2)|N,\mathbf{P}\rangle \\ &\quad= A^{\Gamma}_{NN'}\;2M_{\rm N}N_c\sum_{n,{\rm occ}}\int\!{\rm d}^3\mathbf{X}\;e^{i(\mathbf{P}'-\mathbf{P})\mathbf{X}}\;\bar{\Phi}_n(\mathbf{z}_1-\mathbf{X})\Gamma\Phi_n(\mathbf{z}_2-\mathbf{X})\;e^{iE_n(z_1^0-z_2^0)}+\dots\end{aligned} \tag{6}$$

where for $z_1\neq z_2$ the insertion of the gauge link is understood on the LHS. The dots in Eq. (6) denote terms subleading in the $1/N_c$-expansion, which we will not need here. In Eq. (6) Γ is some Dirac- and flavour-matrix, $A^{\Gamma}_{NN'}$ a constant depending on Γ and the spin and flavour quantum numbers of the nucleon state $|N\rangle = |S_3,T_3\rangle$, and $\Phi_n(\mathbf{x}) = \langle\mathbf{x}|n\rangle$ are the coordinate space wave-functions of the single quark states $|n\rangle$ defined in Eqs. (2,5). The sum in Eq. (6) goes over occupied levels n (i.e. n with $E_n\leq E_{\rm lev}$), and vacuum subtraction is implied for $E_n<E_{\rm lev}$ as in Eq. (4).

Many static nucleonic observables – like magnetic moments, electric polarizabilities, axial properties, etc. – have been computed in the χQSM in the way sketched in Eq. (6). The results were found in good agreement with data, see Ref. [17] for a review. In particular the model describes data on *electromagnetic form factors* up to $|t|\sim 1\,{\rm GeV}^2$ within (10-30)% [16, 17]. In Ref. [18] it has been demonstrated that the model can be applied to the description of *twist 2 quark and anti-quark distribution functions* of the nucleon. The consistency of the approach has been shown by giving *proofs* that the model expressions satisfy all general requirements of QCD [18]. The distribution functions computed in the χQSM [18, 19, 20] refer to a low normalization scale of around 600 MeV, and agree with parameterizations performed at comparably low scales [31] within (10-30)%.

The success of the χQSM in the parameter-free description of – among others – form factors and distribution functions encourages confidence into the predictions for OFDFs made in Refs. [14, 15].

3 Off-forward distribution functions in the χQSM

The unpolarized quark off-forward distribution functions are defined as [3]

$$\begin{aligned}&\int\!\frac{{\rm d}\lambda}{2\pi}\,e^{i\lambda x}\langle\mathbf{P}',S_3'|\bar{\psi}_q(-\lambda n/2)\;\not{n}\;\psi_q(\lambda n/2)|\mathbf{P},S_3\rangle \\ &\qquad= H^q(x,\xi,t)\;\bar{U}(\mathbf{P}',S_3')\not{n}\,U(\mathbf{P},S_3)+E^q(x,\xi,t)\;\bar{U}(\mathbf{P}',S_3')\,\frac{i\sigma^{\mu\nu}n_\mu\Delta_\nu}{2M_{\rm N}}\,U(\mathbf{P},S_3)+\dots\end{aligned} \tag{7}$$

where for brevity the gauge link is omitted and the scale dependence not indicated. The dots denote higher twist contributions. The light-like vector n^μ, the four-momentum transfer Δ^μ, the skewedness parameter ξ and the Mandelstam variable t are defined as

$$n^2 = 0\,,\quad n(P'+P) = 2\,,\quad \Delta^\mu = (P'-P)^\mu\,,\quad n\Delta = -2\xi\,,\quad t = \Delta^2\,. \tag{8}$$

In Eq. (7) $x \in [-1;1]$ with the understanding that for negative x Eq. (7) describes minus the OFDF of the antiquark. When evaluating the above expressions in the χQSM in the large N_c limit, one has to note that $M_{\rm N} = \mathcal{O}(N_c)$ is much larger than spatial components of the momentum transfer $|\Delta^i| = \mathcal{O}(N_c^0)$ which in turn is much larger than the zero-component of the momentum transfer $|\Delta^0| = \mathcal{O}(N_c^{-1})$. The variables x and ξ are of $\mathcal{O}(N_c^{-1})$. Choosing the 3-axis for the light cone space direction, we have in the "large N_c-kinematics"

$$n^\mu = (1,0,0,-1)/M_{\rm N}\,,\quad t = -\boldsymbol{\Delta}^2\,,\quad \xi = -\Delta^3/(2M_{\rm N})\,. \tag{9}$$

Different flavour combinations of the OFDFs exhibit different behaviour in the large N_c limit [7]

$$(H^u+H^d)(x,\xi,t) = N_c^2 f(N_c x, N_c\xi, t)\,,\qquad (E^u-E^d)(x,\xi,t) = N_c^3 f(N_c x, N_c\xi, t)\,, \tag{10}$$
$$(H^u-H^d)(x,\xi,t) = N_c\, f(N_c x, N_c\xi, t)\,,\qquad (E^u+E^d)(x,\xi,t) = N_c^2 f(N_c x, N_c\xi, t)\,. \tag{11}$$

The functions $f(u,v,t)$ in Eqs. (10,11) are stable in the large N_c limit for fixed values of the $\mathcal{O}(N_c^0)$ variables $u = N_c x$, $v = N_c\xi$ and t, and of course different for the different OFDFs.

The model expressions for the leading OFDFs Eq. (10) have been derived in Ref. [14] and read

$$\begin{aligned}(H^u+H^d)(x,\xi,t) \;&=\; M_{\rm N}N_c\int\!{\rm d}^3\mathbf{X}\,e^{i\boldsymbol{\Delta}\mathbf{X}}\sum_{n,{\rm occ}}\int\frac{{\rm d}z^0}{2\pi}\,e^{iz^0(xM_{\rm N}-E_n)}\\ &\quad\times\,\Phi_n^*(\mathbf{X}+\tfrac{z^0}{2}\mathbf{e^3})\,(1+\gamma^0\gamma^3)\,\Phi_n(\mathbf{X}-\tfrac{z^0}{2}\mathbf{e^3})\,,\end{aligned} \tag{12}$$

$$\begin{aligned}(E^u-E^d)(x,\xi,t) \;&=\; \frac{2iM_{\rm N}^2N_c}{3(\boldsymbol{\Delta}^\perp)^2}\int\!{\rm d}^3\mathbf{X}\,e^{i\boldsymbol{\Delta}\mathbf{X}}\sum_{n,{\rm occ}}\int\frac{{\rm d}z^0}{2\pi}\,e^{iz^0(xM_{\rm N}-E_n)}\\ &\quad\times\,\Phi_n^*(\mathbf{X}+\tfrac{z^0}{2}\mathbf{e^3})\,(1+\gamma^0\gamma^3)(\boldsymbol{\tau}\times\boldsymbol{\Delta})^3\Phi_n(\mathbf{X}-\tfrac{z^0}{2}\mathbf{e^3})\,.\end{aligned} \tag{13}$$

Here $\mathbf{e^3}$ is the unit vector of the 3-axis, which is singled out by the choices made in Eq. (9), and $\boldsymbol{\Delta}^\perp$ denotes the part of $\boldsymbol{\Delta}$ transverse to 3-axis.

In Ref. [14] it has been demonstrated that in the forward limit $(H^u+H^d)(x,\xi,t)$, Eq. (12), reduces to the model expression for unpolarized isoscalar distribution function

$$\lim_{\substack{\xi\to 0\\ t\to 0}}(H^u+H^d)(x,\xi,t) = (f_1^u+f_1^d)(x) \tag{14}$$

and that the model expressions Eqs. (12,13) are correctly normalized to the corresponding electromagnetic form factors

$$\int\limits_{-1}^{1}\!{\rm d}x\;(H^u+H^d)(x,\xi,t) = (F_1^u+F_1^d)(t)\;\;,\qquad \int\limits_{-1}^{1}\!{\rm d}x\;(E^u-E^d)(x,\xi,t) = (F_2^u-F_2^d)(t)\;\;. \tag{15}$$

The next and maybe most stringent check of the model expressions is the demonstration that Eqs. (12,13) fulfil the polynomiality condition. As a consequence of Lorentz- and time invariance the $m^{\rm th}$ Mellin moment $M^{(m)}(\xi,t)$ of an OFDF is an even polynomial in ξ of degree m at $t=0$

$$M_H^{q\,(m)}(\xi,0)\equiv\int_{-1}^{1}\mathrm{d}x\,x^{m-1}\,H^q(x,\xi,0)=h_0^{q\,(m)}+h_2^{q\,(m)}\xi^2+\ldots+\begin{cases}h_m^{q\,(m)}\xi^m & \text{for } m \text{ even}\\ h_{m-1}^{q\,(m)}\xi^{m-1} & \text{for } m \text{ odd}\end{cases}\tag{16}$$

$$M_E^{q\,(m)}(\xi,0)\equiv\int_{-1}^{1}\mathrm{d}x\,x^{m-1}\,E^q(x,\xi,0)=e_0^{q\,(m)}+e_2^{q\,(m)}\xi^2+\ldots+\begin{cases}e_m^{q\,(m)}\xi^m & \text{for } m \text{ even}\\ e_{m-1}^{q\,(m)}\xi^{m-1} & \text{for } m \text{ odd,}\end{cases}\tag{17}$$

and due to the spin $\frac{1}{2}$ of the nucleon the highest coefficients of even moments of $H^q(x,\xi,t)$ and $E^q(x,\xi,t)$ are related to each other by

$$h_m^{q\,(m)}=-\,e_m^{q\,(m)}\ .\tag{18}$$

The point $t=0$ is unphysical and can be reached only by means of analytical continuation. In Ref. [14] $(H^u+H^d)(x,\xi,t)$ and $(E^u-E^d)(x,\xi,t)$ have been computed as functions of (physical values of) x, ξ and t, and the polynomiality conditions, Eqs. (16,17), have been checked by taking (numerically) moments $M_H^{(m)}(\xi,t)$, $M_E^{(m)}(\xi,t)$ and extrapolating (numerically) to the unphysical point $t=0$ [24]. It is clear in this way one cannot prove strictly polynomiality. This will be done in the next two sections.

4 Proof of polynomiality for $(H^u+H^d)(x,\xi,t)$

The $m^{\rm th}$ moment of $(H^u+H^d)(x,\xi,t)$ in Eq. (12) – which we will refer to as $M_H^{(m)}(\xi,t)$ – reads (see App. A)

$$M_H^{(m)}(\xi,t)=\frac{N_c}{M_{\rm N}^{m-1}}\sum_{n,{\rm occ}}\sum_{k=0}^{m-1}\binom{m-1}{k}\frac{E_n^{m-1-k}}{2^k}\sum_{j=0}^{k}\binom{k}{j}\langle n|(1+\gamma^0\gamma^3)\,(\hat{p}^3)^j\exp(i\mathbf{\Delta}\hat{\mathbf{X}})\,(\hat{p}^3)^{k-j}|n\rangle\ ,\tag{19}$$

where $\hat{p}^3$, $\hat{\mathbf{X}}$ mean the $3^{\rm rd}$ component of the free-momentum operator and the position operator, respectively, The next step is to take the limit $t\to0$ with $\xi\neq0$ in $M_H^{(m)}(\xi,t)$. Eq. (19) shows that $M_H^{(m)}(\xi,t)=M_H^{(m)}(\mathbf{\Delta})$, i.e. $\mathbf{\Delta}$ determines entirely the dependence of the moments on ξ and t according to Eq. (9). Moreover, $\mathbf{\Delta}$ appears only in the operator $\exp(i\mathbf{\Delta}\hat{\mathbf{X}})$. The order of the operations of taking the limit $t\to0$ and taking matrix elements in Eq. (19) can be interchanged. Taking the limit $t\to0$ (understood as analytical continuation) on $\exp(i\mathbf{\Delta}\hat{\mathbf{X}})$ we obtain (see App. B)

$$\lim_{\substack{\text{analytical}\\ \text{continuation}\\ t\to0}}\exp(i\mathbf{\Delta}\hat{\mathbf{X}})=\sum_{l_e=0}^{\infty}\frac{(-2i\xi M_{\rm N}|\hat{\mathbf{X}}|\,)^{l_e}}{l_e!}\,P_{l_e}(\cos\hat{\theta})\ .\tag{20}$$

Inserting Eq. (20) into Eq. (19) and simplifying the result by using symmetries of the model (see App. C) we obtain

$$\begin{aligned}M_H^{(m)}(\xi,0)\;=\;&\frac{N_c}{M_{\rm N}^{m-1}}\sum_{n,{\rm occ}}\sum_{k=0}^{m-1}\binom{m-1}{k}\frac{E_n^{m-1-k}}{2^k}\sum_{\substack{l_e=0\\ l_e\ \text{even}}}^{\infty}\frac{(-2i\xi M_{\rm N})^{l_e}}{l_e!}\\ &\times\sum_{j=0}^{k}\binom{k}{j}\langle n|(\gamma^0\gamma^3)^k\,(\hat{p}^3)^j|\hat{\mathbf{X}}|^{l_e}\,P_{l_e}(\cos\hat{\theta})\,(\hat{p}^3)^{k-j}|n\rangle\ ,\end{aligned}\tag{21}$$

where $(\gamma^0\gamma^3)^k$, equal to $\gamma^0\gamma^3$ for odd and unity for even k, is introduced to simplify the notation. Note that in the expression for $M_H^{(m)}(\xi,0)$ only even powers of ξ appear. Odd powers of ξ vanish due to parity transformation (see App. C). What remains to be shown is that this series in ξ^2 is a polynomial of degree of less than or equal to m.

The operators in the matrix elements in Eq. (21) transform as irreducible tensor operators $\hat{T}_M^L$ of rank L and $M=0$ under simultaneous rotations in space and isospin-space. More precisely $|\hat{\mathbf{X}}|$ (and any power of it) is rank zero, $\gamma^0\gamma^3$ and $\hat{p}^3$ are rank 1, $P_{l_e}(\cos\hat{\theta})$ is rank l_e (see App. D). The product $[\hat{T}_1]^{L_1}_{M_1}[\hat{T}_2]^{L_2}_{M_2}$ of two irreducible tensor operators $[\hat{T}_1]^{L_1}_{M_1}$ and $[\hat{T}_2]^{L_2}_{M_2}$ can be decomposed into a sum of (certain new) irreducible tensor operators according to

$$[\hat{T}_1]^{L_1}_{M_1}[\hat{T}_2]^{L_2}_{M_2} = \sum_{L=|L_1-L_2|}^{L_1+L_2} \hat{T}_M^L \quad \text{with} \quad \hat{T}_M^L \equiv C(LM;\, L_1M_1, L_2M_2)\, [\hat{T}_1]^{L_1}_{M_1}\, [\hat{T}_2]^{L_2}_{M_2}\,, \tag{22}$$

where the $C(LM;\, L_1M_1, L_2M_2)$ denote Clebsch-Gordan coefficients. The product of three or more irreducible tensor operators can be decomposed in an analogous way.

For even k in Eq. (21), there is a product of $k+1$ irreducible tensor operators, namely the rank-1 operator $\hat{p}^3$, which appears k-times, and the rank-l_e operator $|\hat{\mathbf{X}}|^{l_e}\, P_{l_e}(\cos\hat{\theta})$. For odd k there is the further operator $\gamma^0\gamma^3$, i.e. there are altogether $k+2$ irreducible tensor operators. Upon successive application of Eq. (22) these $k+1$ or $k+2$ irreducible tensor operators yield a series of certain new irreducible tensor operators $[\hat{T}_H]^L_M$ with even ranks L ranging from zero to

$$L_{\max} = \begin{cases} k+l_e+1 & \text{if } k \text{ odd} \\ k+l_e & \text{if } k \text{ even.} \end{cases} \tag{23}$$

Note that only even ranks appear since all involved operators have the "magnetic quantum number" $M=0$. The operators in Eq. (21) can be thus decomposed as

$$(\gamma^0\gamma^3)^k\, (\hat{p}^3)^j |\hat{\mathbf{X}}|^{l_e}\, P_{l_e}(\cos\hat{\theta})\, (\hat{p}^3)^{k-j} = \sum_{L=0}^{L_{\max}} a_L\, [\hat{T}_H]^L_0 \ . \tag{24}$$

Writing out explicitly the dependence of the single quark states $|n\rangle$ on all quantum numbers, Eq. (5), we have

$$\sum_{n,\text{occ}} \langle n|\, (\gamma^0\gamma^3)^k\, (\hat{p}^3)^j |\hat{\mathbf{X}}|^{l_e}\, P_{l_e}(\cos\hat{\theta})\, (\hat{p}^3)^{k-j} |n\rangle$$
$$= \sum_{n,\text{occ}} \sum_{K,M} \sum_{L=0}^{L_{\max}} a_L\, \langle E_n,\, \pi,\, K,\, M|\, [\hat{T}_H]^L_0 |E_n,\, \pi,\, K,\, M\rangle \ . \tag{25}$$

On the RHS of Eq. (25) we recognize traces (sums over matrix elements elements diagonal in K, M) of irreducible tensor operators, and the latter vanish unless the operator has rank zero [32]. As a consequence, the sum over L in Eq. (25) receives a non-zero contribution from $[\hat{T}_H]^0_0$ only.

Consider the case when k is odd. The situation for $L=0$ in which l_e takes its largest possible value $l_e^{\max}$ happens, when $\gamma^0\gamma^3$ and the (in whatever way ordered) k $\hat{p}^3$-operators combine to an operator of rank $k+1$. Then $l_e^{\max}=k+1$. If k is even, $(\gamma^0\gamma^3)^k=1$ does not contribute and then $l_e^{\max}=k$. So, for a given k, the infinite series in l_e in Eq. (21) actually stops at an $l_e^{\max}$ which is a function of k

$$l_e^{\max}(k) = \begin{cases} k+1 & \text{for } k \text{ odd} \\ k & \text{for } k \text{ even.} \end{cases} \tag{26}$$

Inserting the result Eq. (26) into Eq. (21) yields

$$
\begin{aligned}
M_H^{(m)}(\xi,0) \;&=\; \frac{N_c}{M_{\rm N}^{m-1}} \sum_{n,\rm occ} \sum_{k=0}^{m-1} \binom{m-1}{k} \frac{E_n^{m-1-k}}{2^k} \sum_{\substack{l_e=0 \\ l_e\ \rm even}}^{l_e^{\max}(k)} \frac{(-2i\xi M_{\rm N})^{l_e}}{l_e!} \\
&\times \sum_{j=0}^{k} \binom{k}{j} \langle n|(\gamma^0\gamma^3)^k\,(\hat{p}^3)^j |\hat{\mathbf{X}}|^{l_e}\, P_{l_e}(\cos\hat{\theta})\,(\hat{p}^3)^{k-j}|n\rangle\,. \qquad (27)
\end{aligned}
$$

We observe that $M_H^{(m)}(\xi,0)$ is a polynomial in even powers of ξ, the highest power being given by

$$
l_e^{\max}(k)\Big|_{k=m-1} = \begin{cases} m & \text{for } m \text{ even} \\ m-1 & \text{for } m \text{ odd,} \end{cases} \qquad (28)
$$

which completes the proof of polynomiality, Eq. (16), for $(H^u+H^d)(x,\xi,t)$.

5 Proof of polynomiality for $(E^u - E^d)(x,\xi,t)$

The model expression for the $m^{\rm th}$ moment of $(E^u-E^d)(x,\xi,t)$ Eq. (13) is given by (see App. A)

$$
\begin{aligned}
M_E^{(m)}(\xi,t) \;&=\; \frac{2iN_c}{3M_{\rm N}^{m-2}} \sum_{n,\rm occ} \sum_{k=0}^{m-1} \binom{m-1}{k} \frac{E_n^{m-1-k}}{2^k} \sum_{j=0}^{k} \binom{k}{j} \\
&\times \langle n|(1+\gamma^0\gamma^3)\,(\hat{p}^3)^j\,(\boldsymbol{\tau}\times\boldsymbol{\Delta})^3\,\frac{\exp(i\boldsymbol{\Delta}\hat{\mathbf{X}})}{(\boldsymbol{\Delta}^\perp)^2}\,(\hat{p}^3)^{k-j}|n\rangle\,, \qquad (29)
\end{aligned}
$$

where $(\boldsymbol{\tau}\times\boldsymbol{\Delta})^3 \equiv \epsilon^{3kl}\tau^k\Delta^l$. To perform the limit $t\to 0$ in $M_E^{(m)}(\xi,t)$ we have to continue analytically the expression $i\boldsymbol{\Delta}^\perp \exp(i\boldsymbol{\Delta}\hat{\mathbf{X}})/(\boldsymbol{\Delta}^\perp)^2$ to $t=0$. The result reads (see App. B)

$$
\lim_{\substack{\text{analytical} \\ \text{continuation} \\ t\to 0,\,\xi\neq 0}} \frac{i\boldsymbol{\Delta}^\perp \exp(i\boldsymbol{\Delta}\hat{\mathbf{X}})}{(\boldsymbol{\Delta}^\perp)^2} = \sum_{l_e=1}^{\infty} \frac{(-2i\xi M_{\rm N})^{l_e-2}}{l_e!}\; i\Big[\hat{\mathbf{p}}^\perp\;,\;|\hat{\mathbf{X}}|^{l_e}P_{l_e}(\cos\hat{\theta})\Big]\,. \qquad (30)
$$

Inserting Eq. (30) into Eq. (29) and simplifying the result by means of symmetries (see App. C) we obtain

$$
\begin{aligned}
M_E^{(m)}(\xi,0) \;&=\; \frac{2N_c}{3M_{\rm N}^{m-2}} \sum_{n,\rm occ} \sum_{k=0}^{m-1} \binom{m-1}{k} \frac{E_n^{m-1-k}}{2^k} \sum_{j=0}^{k} \binom{k}{j} \sum_{\substack{l_e=2 \\ l_e\ \rm even}}^{\infty} \frac{(-2i\xi M_{\rm N})^{l_e-2}}{l_e!} \\
&\times \langle n|(\gamma^0\gamma^3)^{k+1}\,(\hat{p}^3)^j\; i\Big[(\boldsymbol{\tau}\times\hat{\mathbf{p}})^3\;,\;|\hat{\mathbf{X}}|^{l_e}P_{l_e}(\cos\hat{\theta})\Big]\,(\hat{p}^3)^{k-j}|n\rangle\,. \qquad (31)
\end{aligned}
$$

Note that the contribution of $l_e=1$ in Eq. (30) is singular at $\xi=0$, but it does not contribute in Eq. (31), such that $M_E^{(m)}(\xi,0)$ is well defined for all values of ξ. We observe that $M_E^{(m)}(\xi,0)$ is a series in even powers of ξ. Thus, what remains to be done again, is to demonstrate that this series is actually a polynomial of degree less than or equal to m according to Eq. (17).

For this purpose we note that also $(\boldsymbol{\tau}\times\hat{\mathbf{p}})^3$ is an irreducible tensor operator of rank 1 with $M=0$ (see App. D), such that we deal with matrix elements of a product of irreducible tensor operators in Eq. (31). As argued in the previous section, these operators can be decomposed into a sum of certain new irreducible tensor operators $[\hat{T}_E]^L_M$ with $M = 0$ and (even) ranks L where $0 \leq L \leq L_{\max}$ with

$$L_{\max} = \begin{cases} k + l_e + 1 & \text{if } k \text{ odd} \\ k + l_e + 2 & \text{if } k \text{ even.} \end{cases} \tag{32}$$

The operator in Eq. (31) contains a commutator of $(\boldsymbol{\tau} \times \hat{\mathbf{p}})^3$ and $|\hat{\mathbf{X}}|^{l_e} P_{l_e}(\cos\hat{\theta})$. Focusing first on one ordering of these operators the decomposition into a sum of irreducible tensor operators, which we will call $[\hat{T}_E^{(1)}]^L_M$ looks as follows

$$(\gamma^0\gamma^3)^{k+1}\,(\hat{p}^3)^j\,(\boldsymbol{\tau} \times \hat{\mathbf{p}})^3\,|\hat{\mathbf{X}}|^{l_e}\;P_{l_e}(\cos\hat{\theta})\,(\hat{p}^3)^{k-j} = \sum_{L=0}^{L_{\max}} b_L^{(1)}\;[\hat{T}_E^{(1)}]^L_0\;\;, \tag{33}$$

where the coefficients $b_L^{(1)}$ and the operators $[\hat{T}_E^{(1)}]^L_0$ follow from the successive application of the tensor multiplication rule Eq. (22). However, only the $[\hat{T}_E]^0_0$ contributes to the sum over single quark states in Eq. (31).

There are many ways to construct a rank zero operator out of $(\gamma^0\gamma^3)^{k+1}$, $(\hat{p}^3)^j$, $(\boldsymbol{\tau} \times \hat{\mathbf{p}})^3$, $|\hat{\mathbf{X}}|^{l_e} P_{l_e}(\cos\hat{\theta})$ and $(\hat{p}^3)^{k-j}$. The relevant one (for the proof) is the one which allows l_e to take its maximal value $l_e^{\max}$. This case happens when the ranks of $(\gamma^0\gamma^3)^{k+1}$, $(\hat{p}^3)^j$, $(\boldsymbol{\tau} \times \hat{\mathbf{p}})^3$ and $(\hat{p}^3)^{k-j}$ add up to $k+2$ for even k, or to $k+1$ for odd k. Then $l_e^{\max}(k)$ takes the value

$$l_e^{\max}(k) = \begin{cases} k + 1 & \text{for } k \text{ odd} \\ k + 2 & \text{for } k \text{ even.} \end{cases} \tag{34}$$

Considering the other ordering of $(\boldsymbol{\tau} \times \hat{\mathbf{p}})^3$ and $|\hat{\mathbf{X}}|^{l_e} P_{l_e}(\cos\hat{\theta})$ in the commutator in Eq. (31), one of course arrives at different coefficients $b_L^{(2)}$ and operators $[\hat{T}_E^{(2)}]^L_0$ in Eq. (33), but at the same conclusion Eq. (34). Applying the result Eq. (34) to Eq. (31) we obtain

$$\begin{aligned} M_E^{(m)}(\xi, 0) \;&=\; \frac{2N_c}{3M_{\rm N}^{m-2}} \sum_{n,\mathrm{occ}} \sum_{k=0}^{m-1} \binom{m-1}{k} \frac{E_n^{m-1-k}}{2^k} \sum_{j=0}^{k} \binom{k}{j} \sum_{\substack{l_e=2 \\ l_e\ \text{even}}}^{l_e^{\max}(k)} \frac{(-2i\xi M_{\rm N})^{l_e-2}}{l_e!} \\ &\quad \times \langle n|(\gamma^0\gamma^3)^{k+1}\,(\hat{p}^3)^j\;i\Big[(\boldsymbol{\tau} \times \hat{\mathbf{p}})^3\;,\;|\hat{\mathbf{X}}|^{l_e} P_{l_e}(\cos\hat{\theta})\Big]\,(\hat{p}^3)^{k-j}|n\rangle\;, \end{aligned} \tag{35}$$

and observe that $M_E^{(m)}(\xi, 0)$ is a polynomial in even powers of ξ with the highest power given by

$$l_e^{\max}(k) - 2\Big|_{k=m-1} = \begin{cases} m - 1 & \text{for } m \text{ odd} \\ m - 2 & \text{for } m \text{ even.} \end{cases} \tag{36}$$

The conclusion in Eq. (36) completes our proof of polynomiality, Eq. (17), for $(E^u - E^d)(x, \xi, t)$. Noteworthy is the fact that for an even moment the highest power could be m according to Eq. (17), but it is $m - 2$ in the model. In the next section we will see that this is a consequence of general large N_c counting rules.

6 The *D*-term

The double distributions (DDs) of Refs. [1, 2] have the advantage of satisfying the polynomiality condition automatically. However, the description of non-forward nucleon matrix elements of two-body operators on the light-cone in terms of DDs is incomplete, if one assumes them to be regular functions[2]. This becomes manifest in the absence of the highest power ξ^m in even (singlet) moments

[2] In principle it is possible to obtain a complete description, but only at the prize of introducing singular DDs [22]. The singularities are stronger than a δ-function or any derivative of it [21].

m of $H^q(x,\xi,t)$ and $E^q(x,\xi,t)$. In other words, the coefficients $h_m^{q\,(m)}$ and $e_m^{q\,(m)}$ in Eq. (18) vanish. The description can be, however, completed by introducing the D-term $D^q(z)$, where $z = x/\xi$ [21]. The D-term satisfies $D^q(z) = -D^q(-z)$, has a non-vanishing support for $|z| < 1$ only and is uniquely defined in terms of its moments [21, 22, 23]

$$\int_{-1}^{1} \mathrm{d}z\, z^{m-1} D^q(z) = \begin{cases} h_m^{q\,(m)} = -e_m^{q\,(m)} & \text{for even } m \\ 0 & \text{for odd } m, \end{cases} \tag{37}$$

with $h_m^{q\,(m)}$ and $e_m^{q\,(m)}$ as defined in Eqs. (16,17,18).

The D-term exhibits the following large N_c behaviour [7]

$$\begin{aligned} (D^u + D^d)(z) &= N_c^2 f(z)\ , \\ (D^u - D^d)(z) &= N_c\, f(z)\ , \end{aligned} \tag{38}$$

where the functions $f(z) = \mathcal{O}(N_c^0)$ are constants in the limit of large N_c (and of course different for the different flavour combinations). It is convenient to expand the D-term in terms of Gegenbauer polynomials $C_k^\nu(z)$ for $\nu = \frac{3}{2}$, the solutions of the leading order ERBL-evolution equations [33, 34], which govern its dependence on the renormalization scale μ,

$$D^q(z) = (1 - z^2) \sum_{k=1,3,5,\dots}^{\infty} d_k^q C_k^{3/2}(z)\ . \tag{39}$$

The constant d_1^q is of particular interest, as it determines the behaviour of the D-term (and of OFDFs) in the asymptotic limit of renormalization scale $\mu \to \infty$ [7][3].

In the literature the following phenomenological approach for modeling became popular. One models the regular part of DDs (relying on educated guesses, physical intuition) and supplements the description by adding the D-term (using often information from the χQSM [24]). This approach yields DDs/OFDFs consistent with all general requirements including polynomiality. Many recent estimates and predictions for exclusive processes have been done in this way [22, 24, 35, 36].

The phenomenological interest motivates further studies in the model. Let us discuss first the flavour-non-singlet combination $(D^u - D^d)(z)$. From Eqs. (35,36) we see

$$\int_{-1}^{1} \mathrm{d}z\ z^{m-1} (D^u - D^d)(z) = 0 \quad \forall\, m\ , \tag{40}$$

which means that also $(D^u - D^d)(z) = 0$. Note, however, that this is a zero at the order $\mathcal{O}(N_c^3)$. We obtained this result from moments of $(E^u - E^d)(x,\xi,t) = \mathcal{O}(N_c^3)$ (see Eq. (10)). Due to the relations (18,37) it is $(H^u - H^d)(x,\xi,t) = \mathcal{O}(N_c)$ which dictates the large N_c behaviour of the flavour-non-singlet D-term in Eq. (38), and not $(E^u - E^d)(x,\xi,t) = \mathcal{O}(N_c^3)$. Consequently, the D-term does not contribute to $(E^u - E^d)(x,\xi,t)$ at the order of N_c considered here, which is just our observation in Eq. (40). The result (40) is thus a further demonstration of the consistency of the model, which respects the general large N_c counting rules.

From Eq. (27) we read off the model expressions for moments of the flavour-singlet $(D^u + D^d)(z)$,

$$\begin{aligned} \int_{-1}^{1} \mathrm{d}z\ z^{m-1} (D^u + D^d)(z) &= 2N_c M_{\mathrm{N}} \sum_{n,\,\mathrm{occ}} \langle n|\, \gamma^0 \gamma^3\, \hat{O}^{(m)} |n\rangle\ , \\ \hat{O}^{(m)} &\equiv \frac{i^m}{m!} \sum_{j=0}^{m-1} \binom{m-1}{j} (\hat{p}^3)^j |\hat{\mathbf{X}}|^m\, P_m(\cos\hat{\theta})\, (\hat{p}^3)^{m-j-1}\ , \quad m \text{ even.} \end{aligned} \tag{41}$$

[3] $D^q(z)$ is a singlet quantity and thus mixes under evolution with the corresponding gluon D-term $D^g(z)$. So it is strictly speaking $\sum_q d_1^q + d_1^g$, which is scale independent and determines the asymptotic behaviour of the D-term [7].

Recalling that $M_{\rm N} = \mathcal{O}(N_c)$, we see that all moments of $(D^u + D^d)(z)$ are $\mathcal{O}(N_c^2)$ in the large N_c limit, in agreement with the general counting rules in Eq. (38) also in this case.

In principle Eq. (41) enables us to evaluate all moments of the flavour-singlet D-term. Here we shall restrict ourselves to the evaluation of the $2^{\rm nd}$, i.e. lowest non-vanishing, moment. It is related by $\int_{-1}^{1} {\rm d}z\, z\, D^q(z) = \frac{4}{5}\, d_1^q$ to the first coefficient d_1^q in the Gegenbauer expansion in Eq. (39). The expression for the flavour singlet $d_1 \equiv d_1^u + d_1^d$ follows from Eq. (41) for $m = 2$ and reads

$$d_1 = -\,\frac{5}{4}\, N_c M_{\rm N} \sum_{n,\,{\rm occ}} \langle n|\, \gamma^0\gamma^3 \left\{\hat{p}^3\,,\ |\hat{\mathbf{X}}|\, P_2(\cos\hat{\theta})\right\} |n\rangle\ . \tag{42}$$

We evaluate d_1 by means of the so-called interpolation formula [18], which consists in exactly evaluating the contribution to d_1 Eq. (42) from the discrete level $d_1^{\rm lev}$, and estimating the contribution from the negative Dirac continuum, $d_1^{\rm cont}$, as follows. One rewrites $d_1^{\rm cont}$ in terms of the Feynman quark propagator in the static background pion field U, expands it in a series of gradients of the U-field, and evaluates the leading contribution only. The interpolation formula yields exact results in three limiting cases: (i) low momenta, $|\nabla U| \ll M$, (ii) large momenta, $|\nabla U| \gg M$, (iii) any momenta but small pion field, $|\log U| \ll 1$. One can thus expect that it yields reliable estimates also for the general case. Indeed, it has been observed that estimates obtained in this way agree with results from exact and much more involved numerical calculations within $(10-20)\%$ [18, 19, 20].

The expectation value of the operator in Eq. (42) vanishes in the grandspin $K = 0$ and positive parity state, which characterizes the discrete level, i.e. we have

$$d_1^{\rm lev} = 0\ . \tag{43}$$

For the contribution of $d_1^{\rm cont}$ we obtain

$$d_1^{\rm cont} = -\frac{5}{4}\, f_\pi^2 M_{\rm N} \int\! {\rm d}^3\mathbf{x}\ P_2(\cos\theta)\, \mathbf{x}^2\ {\rm tr}_{\rm F}[\nabla^3 U]\, [\nabla^3 U^\dagger]\ , \tag{44}$$

where ${\rm tr}_{\rm F}$ denotes the trace over flavour. In the calculation leading to Eq. (44) we took the (generally momentum dependent) mass M constant for simplicity. This simplification is legitimate for finite (or at most only logarithmically divergent) quantities, which do not (or only weakly do) depend on regularization.

For constant M the continuum contribution $d_1^{\rm cont}$ is logarithmically divergent and has to be regularized. The same type of divergence appears in the model expression for the pion decay constant f_π. Regularizing both quantities consistently in the same scheme allows to eliminate regulator and cutoff in favour of f_π. It should be noted that further terms, containing three or more gradients of the U-field and neglected in Eq. (44), are finite[4].

In the hedgehog-ansatz the $U(\mathbf{x})$-field is specified by the soliton profile $P(r)$ with $r = |\mathbf{x}|$. We take for $P(r)$ the analytical arctan-profile

$$P(r) = -2{\rm arctan}\frac{R^2}{r^2}\ , \quad R = M^{-1} = (350\,{\rm MeV})^{-1}, \tag{45}$$

which is known to well approximate the self-consistent profile. In Eq. (45) R is the so-called size of the soliton. This enables us to continue the calculation analytically, and yields

$$d_1^{\rm cont} = -\,3\,\pi^2\sqrt{2}\ f_\pi^2 M_{\rm N} R^3\ . \tag{46}$$

With $f_\pi = 91\,{\rm MeV}$ and the value $M_{\rm N} = 1170$ MeV obtained in the χQSM with the analytical arctan-profile Eq. (45) – and "adding up" the zero Eq. (43) – we obtain

$$d_1 = -9.46\ \ . \tag{47}$$

[4]For more technical details on the interpolation formula, and examples of similar calculations see Refs. [18, 19].

In order to understand how this result can be used for phenomenology, we write down the entire sum rule for the second moment of $(H^u + H^d)(x,\xi,t)$ which reads

$$\int_{-1}^{1} \mathrm{d}x\, x\, (H^u + H^d)(x,\xi,0) = M_Q + \frac{4}{5}\, d_1\, \xi^2\ , \tag{48}$$

where M_Q is the fraction of the nucleon momentum carried by quark and antiquarks. In nature one observes $M_Q \simeq 0.5$ already at moderates scales of few GeV^2. In the χQSM, at its low scale of around $0.4\,\mathrm{GeV}^2$, M_Q is exactly unity (as it has to be since there are no gluons in the model and the entire nucleon momentum has to be carried by quarks and antiquarks alone). It is more reliable to consider ratios of model quantities than absolute numbers. Therefore we identify

$$\left[\frac{d_1}{M_Q}\right]_{\substack{\text{model}\\ \text{low scale}}} \simeq \left[\frac{d_1}{M_Q}\right]_{\substack{\text{nature}\\ \text{few GeV}^2}} . \tag{49}$$

The relation (49) is only approximate, since the ratio d_1/M_Q is strictly speaking scale dependent. However, the effects of evolution are smaller than the generic accuracy of the model results, and we neglect them here. With $M_Q \simeq 0.5$ at experimentally relevant scales of few GeV^2 we obtain the following prediction from the χQSM,

$$d_1 \simeq -4.7 \ \text{ at few GeV}^2. \tag{50}$$

The result Eq. (50) is very close to the number $d_1 \simeq -4.0$, which has been extracted in Ref. [24]. In Ref. [24] the moments $M_H^{(m)}(\xi,t)$ of $(H^u + H^d)(x,\xi,t)$ from Ref. [14] have been computed for physical values of t, and then extrapolated numerically to $t = 0$. The result Eq. (50) and that of Ref. [24] have been obtained using the same procedure, but with different regularizations. In Ref. [14] $(H^u+H^d)(x,\xi,t)$ has been regularized with the momentum dependent mass $M(p)$. Here we took constant M (and applied some regularization scheme, the dependence on which has been then removed in favour of f_π). This explains the small difference between the two numbers. Keeping this in mind, our result Eq. (50) well reproduces and confirms the numerical result obtained in Ref. [24].

The advantage of our method is that we are in position to exactly evaluate, at least in principle, any moment of the D-term in the model. In Ref. [24] this was not possible, not even in principle, due to limitations set by numerical accuracy.

7 Summary and conclusions

We have presented explicit proofs that the χQSM expressions for the leading flavour combinations in the large N_c limit of the unpolarized OFDFs $(H^u + H^d)(x,\xi,t)$ and $(E^u - E^d)(x,\xi,t)$ satisfy the polynomiality condition. The method can be strait forwardly generalized to the case of flavour combinations subleading in N_c, and to the case of helicity dependent OFDFs. The proof is an important contribution to the demonstration of the theoretical consistency of the description of OFDFs in the framework of the χQSM, and increases the credibility and reliability of theoretical predictions made on the basis of the χQSM.

As a byproduct of the proof, we derived explicit model expressions for the moments of the flavour-singlet D-term, which are of phenomenological and theoretical interest. The explicit model expressions not only shed some light on the chiral dynamics underlying the D-term, but also enable exact model calculations of the coefficients in the Gegenbauer expansion of the D-term. We have demonstrated it by analytically computing the first coefficient d_1 for the flavour-singlet

D-term reproducing the numerically estimated value previously obtained. Calculations of higher Gegenbauer coefficients are in progress.

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A Mellin moments of $(H^u + H^d)$ and $(E^u - E^d)$

$H(x,\xi,t) \equiv (H^u + H^d)(x,\xi,t)$ and $E(x,\xi,t) \equiv (E^u - E^d)(x,\xi,t)$, Eqs. (12,13), can be written compactly as

$$A(x,\xi,t) = c_A \sum_{n\,\mathrm{occ}} \int \frac{\mathrm{d}z^0}{2\pi}\, e^{iz^0(xM_\mathrm{N}-E_n)}\, f_n(\Gamma_A, t, \mathbf{\Delta}) \tag{51}$$

with

$$\begin{aligned} c_H &= N_c M_\mathrm{N}\;, \qquad , \qquad & c_E &= (2/3) i N_c M_\mathrm{N}^2/(\mathbf{\Delta}^\perp)^2\;, \\ \Gamma_H &= (1+\gamma^0\gamma^3)\;, & \Gamma_E &= (1+\gamma^0\gamma^3)(\boldsymbol{\tau}\times\mathbf{\Delta})^3\;, \end{aligned} \tag{52}$$

and $f_n(\Gamma, t, \mathbf{\Delta})$ defined as

$$\begin{aligned} f_n(\Gamma, z^0, \mathbf{\Delta}) &\equiv \int \mathrm{d}^3\mathbf{X}\, e^{i\mathbf{\Delta X}} \Phi_n^*(\mathbf{X} + \frac{z^0}{2}\mathbf{e^3})\,\Gamma\Phi_n(\mathbf{X} - \frac{z^0}{2}\mathbf{e^3}) \\ &= \langle n|\,\Gamma\, \exp(-i\frac{z^0}{2}\hat{p}^3)\, \exp(i\mathbf{\Delta}\hat{\mathbf{X}})\, \exp(-i\frac{z^0}{2}\hat{p}^3)|n\rangle \end{aligned} \tag{53}$$

where in the second line we made use of $\Phi_n(\mathbf{X}-\mathbf{a}) = \langle \mathbf{X}-\mathbf{a}|n\rangle = \langle \mathbf{X}|\, \exp(-i\mathbf{a}\hat{\mathbf{p}})|n\rangle$ and analogously for $\Phi_n^*(\mathbf{X}-\mathbf{a})$. For the m^{th} moment $M_A^{(m)}(\xi,t)$ we obtain

$$\begin{aligned} M_A^{(m)}(\xi,t) &\equiv \int_{-1}^{1}\mathrm{d}x\; x^{m-1}\; A(x,\xi,t) = c_A \sum_{n\,\mathrm{occ}} \int_{-1}^{1}\mathrm{d}x\; x^{m-1} \int\frac{\mathrm{d}z^0}{2\pi}\, e^{iz^0(xM_\mathrm{N}-E_n)}\, f_n(\Gamma_A, z^0, \mathbf{\Delta}) \\ &= \frac{c_A}{M_\mathrm{N}^m}\sum_{n\,\mathrm{occ}} \int\frac{\mathrm{d}z^0}{2\pi}\left[\frac{\partial^{m-1}}{\partial(iz^0)^{m-1}} \int_{-\infty}^{\infty}\mathrm{d}y\; e^{iz^0 y}\right] e^{-iz^0E_n}\, f_n(\Gamma_A, z^0, \mathbf{\Delta}) \\ &= \frac{c_A}{M_\mathrm{N}^m}\sum_{n\,\mathrm{occ}} \left[\frac{\partial^{m-1}}{\partial(-iz^0)^{m-1}} e^{-iz^0E_n} f_n(\Gamma_A, z^0, \mathbf{\Delta})\right]_{z^0=0} \\ &= \frac{c_A}{M_\mathrm{N}^m}\sum_{n\,\mathrm{occ}} \sum_{k=0}^{m-1}\binom{m-1}{k} E_n^{m-1-k} \left[\frac{\partial^k f_n(\Gamma_A, z^0, \mathbf{\Delta})}{\partial(-iz^0)^k}\right]_{z^0=0}\;. \end{aligned} \tag{54}$$

In the second line of Eq. (54), after the substitution $y = xM_\mathrm{N}$, the new integration limits $[-M_\mathrm{N},\, M_\mathrm{N}]$ have been replaced by $[-\infty, \infty]$. This step is justified in large N_c limit where $M_\mathrm{N} = \mathcal{O}(N_c)$. Differentiating $f_n(\Gamma, z^0, \mathbf{\Delta})$ Eq. (53) k-times with respect to $(-iz^0)$ and setting then $z^0 = 0$ yields

$$\left[\frac{\partial^k f_n(\Gamma, z^0, \mathbf{\Delta})}{\partial(-iz^0)^k}\right]_{z^0=0} = \frac{1}{2^k}\sum_{j=0}^{k}\binom{k}{j} \langle n|\Gamma\,(\hat{p}^3)^j \exp(i\mathbf{\Delta}\hat{\mathbf{X}})\,(\hat{p}^3)^{k-j}|n\rangle\;. \tag{55}$$

Inserting Eq. (55) into Eq. (54) we obtain

$$M_A^{(m)}(\xi,t) = \frac{c_A}{M_\mathrm{N}^m}\sum_{n,\mathrm{occ}} \sum_{k=0}^{m-1}\binom{m-1}{k} \frac{E_n^{m-1-k}}{2^k} \sum_{j=0}^{k}\binom{k}{j} \langle n|\Gamma_A\,(\hat{p}^3)^j \exp(i\mathbf{\Delta}\hat{\mathbf{X}})\,(\hat{p}^3)^{k-j}|n\rangle\;. \tag{56}$$

which is the result quoted in Eq. (19) and in Eq. (29), respectively.

B Analytical continuation to $t = 0$

The moments $M_H(\xi,t)$ and $M_E(\xi,t)$ in Eq. (56) depend on the variables ξ and t only through the 3-momentum transfer $\mathbf{\Delta}$ in the "large N_c kinematics" Eq. (9). As only ξ and t are relevant variables, $\mathbf{\Delta}$ can be expressed through ξ, t and some arbitrary real angle α as

$$\mathbf{\Delta} = \left(-\sin\alpha\sqrt{-t-(2\xi M_{\rm N})^2}\;,\;\cos\alpha\sqrt{-t-(2\xi M_{\rm N})^2}\;,\;(2\xi M_{\rm N})\right)\,. \tag{57}$$

The arbitrariness of α reflects the azimuthal asymmetry of the physical situation around the 3-axis chosen for the space direction of the light-cone in Eq. (9). Since there is no real dependence on this angle, we can choose a particular value for α, or equivalently average over it.

(i) $\boldsymbol{(H^u+H^d)}$**.** To continue the moments $M_H(\xi,t)$ to $t=0$, we have to continue analytically the function $F_H(\xi,t)\equiv\exp(i\mathbf{\Delta X})$ in Eq. (19). Taking $\mathbf{\Delta}$ as defined in Eq. (57) we obtain a function $F_H(\xi,t)$ depending in addition on the angle α, and we remove the dependence on this arbitrary angle by averaging over $\alpha\in[0,2\pi]$. Taking $\mathbf{X}=|\mathbf{X}|(\sin\theta\cos\phi,\,\sin\theta\sin\phi,\,\cos\theta)$ in spherical coordinates and expanding $\exp(i\mathbf{\Delta X})$ in a series, we obtain for $F_H(\xi,t)$

$$F_H(\xi,t)\equiv\exp(i\mathbf{\Delta X})=\sum_{l=0}^{\infty}\frac{(-i|\mathbf{X}|)^l}{l!}\,\frac{1}{2\pi}\int_0^{2\pi}\!{\rm d}\alpha\left(\sqrt{-t-(2\xi M_{\rm N})^2}\,\sin\theta\sin(\alpha-\phi)+(2\xi M_{\rm N})\,\cos\theta\right)^l\,. \tag{58}$$

We see that $F_H(\xi,t)$ is an analytic function of the variables ξ and t everywhere except for the twofold branching point $t=-(2\xi M_{\rm N})^2$. Taking the limit $t\to 0$ within a definite Riemann sheet, we obtain

$$\lim_{\substack{\text{analytical}\\ \text{continuation}\\ t\to 0}} F_H(\xi,t)=\sum_{l=0}^{\infty}\frac{(-i2\xi M_{\rm N}|\mathbf{X}|)^l}{l!}\,\frac{1}{2\pi}\int_0^{2\pi}\!{\rm d}\alpha\left(\pm i\,\sin\theta\sin(\alpha-\phi)+\cos\theta\right)^l \tag{59}$$

with the plus and minus signs refering to the two Riemann sheets. The final result is real and does not depend on the choice of sheet

$$\frac{1}{2\pi}\int_0^{2\pi}\!{\rm d}\alpha\left(\pm i\sin\theta\sin(\alpha-\phi)+\cos\theta\right)^l=\frac{1}{\pi}\int_0^{\pi}\!{\rm d}\alpha\left(\pm i\sin\theta\sin\alpha+\cos\theta\right)^l=P_l(\cos\theta)\,. \tag{60}$$

One arrives at the intermediate result in Eq. (60) by using the periodicity of the sine and cosine functions. The last equality is the *integral representation of Laplace and Mehler for Legendre polynomials* [37]. Inserting Eq. (60) in Eq. (59) we arrive at the result quoted in Eq. (20),

$$\lim_{\substack{\text{analytical}\\ \text{continuation}\\ t\to 0}} F_H(\xi,t)=\sum_{l=0}^{\infty}\frac{(-i2\xi M_{\rm N}|\mathbf{X}|)^l}{l!}\,P_l(\cos\theta)\,. \tag{61}$$

(ii) $\boldsymbol{(E^u-E^d)}$**.** From Eq. (29) we see, that the function $F_E(\xi,t)$ – which is to be continued to the point $t=0$ in that case – reads

$$F_E(\xi,t)\equiv\frac{i\mathbf{\Delta}^\perp\exp(i\mathbf{\Delta X})}{(\mathbf{\Delta}^\perp)^2}=\mathbf{\nabla}^\perp\,\frac{F_H(\xi,t)}{-t-(2\xi M_{\rm N})^2} \tag{62}$$

with $F_H(\xi,t)$ from Eq. (58). In Eq. (62) $\boldsymbol{\nabla}^\perp \equiv (\frac{\partial}{\partial X^1}, \frac{\partial}{\partial X^2})$. For $\xi \neq 0$ the limit $t \to 0$ yields

$$\lim_{\substack{\text{analytical}\\ \text{continuation}\\ t\to 0,\ \xi\neq 0}} F_E(\xi,t) = \boldsymbol{\nabla}^\perp \sum_{l=1}^{\infty} \frac{(-i2\xi M_{\rm N})^{l-2}}{l!} P_l(\cos\theta) \ , \tag{63}$$

where we took the limit independently on numerator and denominator, since both limits exist, and used the result (61). The $l = 0$ addend in the sum in Eq. (63) is a constant and vanishes upon differentiation, so the sum starts with $l = 1$. In the next Appendix we will see that the addend for $l = 1$, which is singular at $\xi = 0$, does not contribute. Thus the moments $M_E^{(m)}(\xi, 0)$ are well defined for all ξ.

Using the identity $\frac{\partial}{\partial \hat{\mathbf{X}}} F(\hat{\mathbf{X}}) = i[\hat{\mathbf{p}}, F(\hat{\mathbf{X}})]$ in Eq. (63) we obtain the result quoted in Eq. (30).

C Symmetries of the model

***G_5*-symmetry.** Consider the unitary matrix G_5 with the following property

$$G_5\gamma^\mu G_5^{-1} = (\gamma^\mu)^T \ \ , \ \ G_5\tau^a G_5^{-1} = -(\tau^a)^T \ . \tag{64}$$

In the standard representation of γ- and τ-matrices $G_5 = \gamma^1\gamma^3\tau^2$. Noting that $\nabla^i = -(\nabla^i)^T$ holds formally, one finds that G_5 transforms the Hamiltonian $\hat{H}_{\rm eff}$ and the single quark states, Eqs. (2,5), in coordinate space representation as $G_5\hat{H}_{\rm eff}G_5^{-1} = \hat{H}_{\rm eff}^T$ and $G_5\Phi_n(\mathbf{x}) = \Phi_n^*(\mathbf{x})$ [17].

Let Γ be some matrix in Dirac- and flavour-space and define $\tilde{\Gamma} \equiv (G_5\Gamma G_5^{-1})^T$. Let $F(\hat{\mathbf{X}})$ be a function $F(\hat{\mathbf{X}})$ of $\hat{\mathbf{X}}$. Then

$$\langle n|\Gamma(\hat{p}^3)^l F(\hat{\mathbf{X}})(\hat{p}^3)^m|n\rangle = (-1)^{l+m}\langle n|\tilde{\Gamma}(\hat{p}^3)^m F(\hat{\mathbf{X}})(\hat{p}^3)^l|n\rangle \ \ . \tag{65}$$

Parity. The parity operator $\hat{\Pi} = \hat{\Pi}^{-1} \equiv \gamma^0\hat{\mathcal{P}}$, where $\hat{\mathcal{P}}F(\hat{\mathbf{X}})\hat{\mathcal{P}}^{-1} = F(-\hat{\mathbf{X}})$, acts on the single quark states as $\hat{\Pi}|n\rangle = \pm|n\rangle$ according to the parity of the single quark state Eq. (5). With the notation of Eq. (65) we have upon application of the parity transformation

$$\langle n|\Gamma(\hat{p}^3)^l F(\hat{\mathbf{X}})(\hat{p}^3)^m|n\rangle = (-1)^{l+m}\langle n|\left(\gamma^0\Gamma\gamma^0\right)(\hat{p}^3)^l\left(\hat{\mathcal{P}}F(\hat{\mathbf{X}})\hat{\mathcal{P}}^{-1}\right)(\hat{p}^3)^m|n\rangle \ . \tag{66}$$

***(i)* $(H^u{+}H^d)$.** Here $\Gamma = (1{+}\gamma^0\gamma^3)$ and $\tilde{\Gamma} = (1{-}\gamma^0\gamma^3)$, and $F(\hat{\mathbf{X}}) = |\hat{\mathbf{X}}|^{l_e}\hat{P}_{l_e}$ where $\hat{P}_l \equiv P_l(\cos\hat{\theta})$ is set for brevity. According to Eq. (65)

$$\sum_{j=0}^{k}\binom{k}{j}\langle n|(1+\gamma^0\gamma^3)(\hat{p}^3)^j|\hat{\mathbf{X}}|^l\hat{P}_l(\hat{p}^3)^{k-j}|n\rangle = (-1)^k\sum_{j=0}^{k}\binom{k}{j}\langle n|(1-\gamma^0\gamma^3)\,(\hat{p}^3)^{k-j}|\hat{\mathbf{X}}|^l\hat{P}_l\,(\hat{p}^3)^j|n\rangle \ . \tag{67}$$

From Eq. (67) follows that the contribution of the "1" in $(1+\gamma^0\gamma^3)$ vanishes for odd k, while the contribution of "$\gamma^0\gamma^3$" vanishes for even k. Using $(\gamma^0\gamma^3)^k = 1$ for even k, and $(\gamma^0\gamma^3)^k = \gamma^0\gamma^3$ for odd k, this conclusion can be written compactly as

$$\sum_{j=0}^{k}\binom{k}{j}\langle n|(1+\gamma^0\gamma^3)(\hat{p}^3)^j|\hat{\mathbf{X}}|^l\hat{P}_l\,(\hat{p}^3)^{k-j}|n\rangle = \sum_{j=0}^{k}\binom{k}{j}\langle n|(\gamma^0\gamma^3)^k(\hat{p}^3)^j|\hat{\mathbf{X}}|^l\hat{P}_l\,(\hat{p}^3)^{k-j}|n\rangle \ . \tag{68}$$

Applying Eq. (66) to the intermediate result Eq. (68) yields

$$\sum_{j=0}^{k}\binom{k}{j}\langle n|\,(\gamma^0\gamma^3)^k\,(\hat{p}^3)^j|\hat{\mathbf{X}}|^l\hat{P}_l\,(\hat{p}^3)^{k-j}|n\rangle = (-1)^l\sum_{j=0}^{k}\binom{k}{j}\langle n|\,(\gamma^0\gamma^3)^k\,(\hat{p}^3)^j|\hat{\mathbf{X}}|^l\hat{P}_l\,(\hat{p}^3)^{k-j}|n\rangle \ , \tag{69}$$

i.e. the expression vanishes if l is odd. The result Eq. (69) yields the expression in Eq. (21).

(ii) $(\boldsymbol{E^u - E^d})$**.** Here $\Gamma = (1+\gamma^0\gamma^3)\tau^a$ with $\tilde{\Gamma} = (-1+\gamma^0\gamma^3)\tau^a$. It is convenient to take $F(\hat{\mathbf{X}})$ as in Eq. (63), $F(\hat{\mathbf{X}}) = (\frac{\partial}{\partial \hat{X}^b}|\hat{\mathbf{X}}|^l \hat{P}_l) \equiv (\hat{\nabla}^b|\hat{\mathbf{X}}|^l\hat{P}_l)$. Note that the derivative acts on $|\hat{\mathbf{X}}|^l\hat{P}_l$ only. The symmetry transformations Eqs. (65,66) yield

$$\sum_{j=0}^{k}\binom{k}{j}\langle n|(\gamma^0\gamma^3)^{k+1}\tau^a\,(\hat{p}^3)^j(\hat{\nabla}^b|\hat{\mathbf{X}}|^l\hat{P}_l)(\hat{p}^3)^{k-j}|n\rangle$$
$$= (-1)^l\sum_{j=0}^{k}\binom{k}{j}\langle n|(\gamma^0\gamma^3)^{k+1}\tau^a\,(\hat{p}^3)^j(\hat{\nabla}^b|\hat{\mathbf{X}}|^l\hat{P}_l)(\hat{p}^3)^{k-j}|n\rangle\ . \tag{70}$$

I.e. also here contributions of odd l vanish. Applying Eq. (70) to $M_E^{(m)}(\xi,0)$ and using the identity $\hat{\nabla}^b F(\hat{\mathbf{X}}) = i[\hat{p}^b, F(\hat{\mathbf{X}})]$ yields the result quoted in Eq. (31).

D Irreducible tensor operators

The grand spin operator is defined as

$$\hat{\mathbf{K}} = \hat{\mathbf{L}} + \hat{\mathbf{S}} + \hat{\mathbf{T}} \tag{71}$$

where $\hat{\mathbf{L}} = \hat{\mathbf{r}}\times\hat{\mathbf{p}}$ is the orbital angular momentum operator, $\hat{\mathbf{S}} = \frac{1}{2}\gamma_5\gamma^0\boldsymbol{\gamma}$ is the spin operator and $\hat{\mathbf{T}} = \frac{1}{2}\boldsymbol{\tau}$ is the isospin operator. $\hat{\mathbf{K}}$ is the generator of simultaneous rotations R in space and $SU(2)$-flavour space. Let $\mathbf{n}$ be a unit vector defining an axis in space and flavour space and let α be an angle. The rotation R is then characterized by the unitary operator $\hat{U}(R)$

$$\hat{U}(R) = \hat{U}(\mathbf{n},\alpha) = \exp(-i\,\alpha\,\mathbf{n}\cdot\hat{\mathbf{K}})\ . \tag{72}$$

The unitary transformation Eq. (72) transforms single quark states into "rotated quark states" according to

$$\hat{U}(R)|E_n,\pi,K,M\rangle = \sum_{M'=-K}^{K}|E_n,\pi,K,M'\rangle D^{(K)}_{M'M}(R)\ .$$

Here $D^{(K)}_{M'M}(R)$ denote finite rotation Wigner matrices.

The operators $\hat{T}^L_M$ with integers L, $M\in[-L,\ ,L]$ are said to be irreducible tensor operators of rank L, if they transform into each other under the action of the unitary transformation Eq. (72) according to

$$\hat{U}(R)\,\hat{T}^L_M\,\hat{U}^\dagger(R) = \sum_{M'=-L}^{L} D^{(L)}_{M'M}(R)\,\hat{T}^L_{M'}\ . \tag{73}$$

By considering infinitesimal rotations one arrives at a definition, which is equivalent to Eq. (73)

$$\begin{aligned}\left[\hat{K}^3\,,\,\hat{T}^L_M\right] &= M\,\hat{T}^L_M\\ \left[\hat{K}^\pm,\hat{T}^L_M\right] &= \sqrt{L(L+1)-M(M\pm1)}\ \hat{T}^L_{M\pm1}\quad\text{where}\quad \hat{K}^\pm\equiv\hat{K}^1\pm i\hat{K}^2\ ,\end{aligned} \tag{74}$$

but allows more easily to check whether a given operator is an irreducible tensor operator or not. The Hamiltonian $\hat{H}_{\rm eff}$ transforms as $\hat{U}(R)\hat{H}_{\rm eff}\hat{U}^{-1}(R) = \hat{H}_{\rm eff}$, since it commutes with $\hat{\mathbf{K}}$. This means that $\hat{H}_{\rm eff}$ is an irreducible tensor operator of rank zero.

Further examples of rank zero operators are $|\hat{\mathbf{X}}|$, $\boldsymbol{\tau}\cdot\hat{\mathbf{X}}$ or $\gamma^0\boldsymbol{\gamma}\cdot\hat{\mathbf{p}}$, which satisfy Eqs. (74) for $L = M = 0$. The operators

$$
\begin{aligned}
[\hat{T}_a]^1_M &= \begin{cases} i(\mp\hat{p}^1 - i\hat{p}^2)/\sqrt{2} & \text{for } M = \pm 1 \\ i\hat{p}^3 & \text{for } M = 0, \end{cases} \\
[\hat{T}_b]^1_M &= \begin{cases} i(\mp\tau^1 - i\tau^2)/\sqrt{2} & \text{for } M = \pm 1 \\ i\tau^3 & \text{for } M = 0, \end{cases} \\
[\hat{T}_c]^1_M &= \begin{cases} i(\mp\hat{X}^1 - i\hat{X}^2)/\sqrt{2} & \text{for } M = \pm 1 \\ i\hat{X}^3 & \text{for } M = 0, \end{cases} \\
[\hat{T}_d]^1_M &= \begin{cases} i(\mp\gamma^0\gamma^1 - i\gamma^0\gamma^2)/\sqrt{2} & \text{for } M = \pm 1 \\ i\gamma^0\gamma^3 & \text{for } M = 0, \end{cases} \\
[\hat{T}_e]^1_M &= \begin{cases} i(\mp(\boldsymbol{\tau}\times\hat{\mathbf{p}})^1 - i(\boldsymbol{\tau}\times\hat{\mathbf{p}})^2)/\sqrt{2} & \text{for } M = \pm 1 \\ i(\boldsymbol{\tau}\times\hat{\mathbf{p}})^3 & \text{for } M = 0 \end{cases}
\end{aligned} \tag{75}
$$

are irreducible tensor operators of rank 1 since they satisfy Eqs. (74) for $L = 1$.

Finally Legendre polynomials $P_l(\cos\hat{\theta})$ are irreducible spherical tensor operators $\hat{T}^l_m$ with $m = 0$, since $P_l(\cos\hat{\theta}) \propto Y^l_m(\hat{\Omega})|_{m=0} \equiv \hat{T}^l_m|_{m=0}$. The spherical harmonics $Y^l_m(\hat{\Omega})$ are irreducible tensor operators of rank l, since they satisfy Eqs. (74).

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